

Write your homework *neatly, in pencil*, on blank white $8\frac{1}{2} \times 11$ printer paper. Always *write the problem*, or at least enough of it so that your work is readable. If the problem involves a function, write the function. If the problem involves an equation, write the equation. Use words, and when appropriate, *write in sentences*.

Definition 1. Let $a \in (0, 1) \cup (1, \infty)$. Define the *natural and base a logarithm and exponential* functions thusly:

- $\log : (0, \infty) \rightarrow \mathbb{R}$ given by $\log(x) = \int_1^x \frac{1}{t} dt$
- $\exp : \mathbb{R} \rightarrow (0, \infty)$ such that $\exp(x) = y \Leftrightarrow x = \log(y)$
- $\exp_a : \mathbb{R} \rightarrow (0, \infty)$ given by $\exp_a(x) = \exp(x \log(a))$
- $\log_a : (0, \infty) \rightarrow \mathbb{R}$ such that $\log_a(x) = y \Leftrightarrow x = \exp_a(y)$

Define the number e and a^x by

- $e = \exp(1)$
- $a^x = \exp_a(x)$ whence $e^x = \exp(x)$

We have shown that

- $\frac{d}{dx} \log(x) = \frac{1}{x}$
- $\frac{d}{dx} \exp(x) = \exp(x)$
- $\frac{d}{dx} \exp_a(x) = \log(a) \exp_a(x)$
- $\frac{d}{dx} \log_a(x) = \frac{1}{\log(a)x}$

Problem 1 (Thomas §7.4 # 15). Compute $\frac{dy}{dx}$ where

- (a) $y = x^\pi$
- (b) $y = \pi^x$

Problem 2 (Thomas §7.4 # 21). Find $\frac{dy}{dx}$ where

$$y = 2^{\sin 3t}.$$

Problem 3 (Thomas §7.4 # 43). Find $\frac{dy}{dx}$ where

$$y = (\sin x)^x.$$

Problem 4 (Thomas §7.4 # 51). Evaluate the definite integral

$$\int_1^{\sqrt{2}} x 2^{(x^2)} dx.$$

Problem 5 (Thomas §7.4 # 69). Evaluate the indefinite integral

$$\int \frac{dx}{x \log_{10} x}.$$

Problem 6 (Thomas §7.4 # 90a). Find the equation of the line through the origin and tangent to the graph of $y = \ln(x)$.

Problem 7 (Thomas §7.4 # 90b). Show that $\ln(x^e) < x$ for all positive $x \neq e$.

Problem 8 (Thomas §7.4 # 90c). Show that $x^e < e^x$ for all positive $x \neq e$. Conclude that $\pi^e < e^\pi$.

It may be helpful to use this “defining property”:

$$\log_b(x) = y \quad \Leftrightarrow \quad b^y = x.$$

Problem 9. Evaluate.

(a) $\log_3 81$

(b) $\log_{32} 8$

(c) $\log_3 162 - \log_3 2$

(d) $\log_5 \sqrt[3]{625}$

(e) $\log_{10} \frac{32}{5} - \log_{10} \frac{16}{25}$

Problem 10. Solve.

(a) $27^{5x-6} = 81^{2x+11}$

(b) $\log_{13} x = 2$

(c) $\log_2(x+5) + \log_2(x+1) = 3 + \log_2(x-1)$

(d) $1331^{(2x+1)} = \frac{1}{121^{(x-5)}}$

(e) $\log_x(x-2) + \log_x(x-6) = 2$